

The smallest neutrino mass

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Abstract

We consider models where one Majorana neutrino is massless at tree level (like the see saw with two right-handed neutrinos), and compute the contribution to its mass m generated by two-loop quantum corrections. The result is $m \sim 10^{-13}$ eV in the SM and $m \sim 10^{-10}$ eV $\cdot (\tan \beta/10)^4$ in the MSSM, compatible with the restricted range suggested by Affleck-Dine baryogenesis.

1 Introduction

Oscillation data [1] demand that two neutrinos are massive and strongly mixed; in particular a roughly $\nu_\mu + \nu_\tau$ mass eigenstate is demanded by the atmospheric anomaly. The third neutrino mass eigenstate might be massless, and this possibility is realized in various theoretical models, such as see-saw models with two right-handed neutrinos [2]. Our study applies to generic models, where lepton number is broken at some high scale leaving Majorana masses for two neutrinos. Since e, μ, τ have different Yukawa couplings, no symmetry demands that the massless neutrino stays massless: with the inclusion of quantum corrections all neutrinos become massive. In section 2 we compute the neutrino mass generated by renormalization-group equation (RGE) effects in the Standard Model (i.e. without new degrees of freedom up to the scale where neutrino masses are generated). In section 3 we consider the Minimal Supersymmetric Standard Model (MSSM), and in section 4 we show that this quantum correction could allow successful Affleck-Dine (AD) baryogenesis via leptogenesis along the LH_u flat direction [3].

2 Standard Model

Within the Standard Model (SM), Majorana neutrino masses are described by the effective operator $(L_i H)(L_j H)$ where L and H are the lepton and Higgs doublets. Its coefficients can be parameterized by the neutrino mass matrix m_{ij} . The dominant effect that increases the rank of m_{ij} is the two-loop diagram shown in fig. 1 (left). See [4] for earlier related studies. The effect is conveniently described in terms of the RGE for m :

$$(4\pi)^2 \frac{dm}{d \ln \mu} = m(\lambda - 3g_2^2 + 6\lambda_t^2) - \frac{3}{2}(m \cdot Y^T + Y \cdot m) + \frac{2}{(4\pi)^2} Y \cdot m \cdot Y^T + \dots \quad (1)$$

Here m and $Y = \lambda_E^\dagger \cdot \lambda_E$ are 3×3 matrices in flavour space (λ_E is the charged-lepton Yukawa coupling), and λ , g_2 and λ_t denote the Higgs self-coupling, the $SU(2)_L$ gauge coupling and

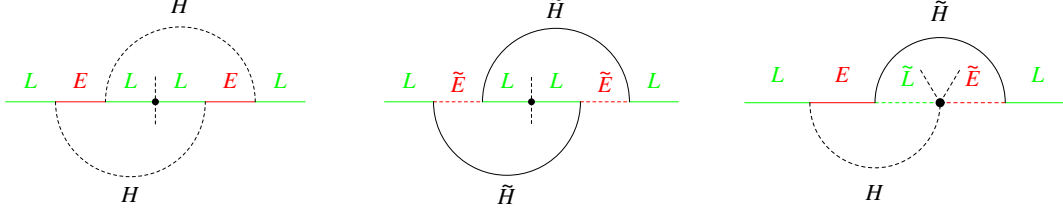


Figure 1: *Feynman diagrams (in the $SU(2)$ -symmetric limit) that increase the rank of the neutrino mass matrix, in the SM (left diagram) and in the MSSM. The black dot denotes the effective operator generating the Majorana mass (in the MSSM case we omitted diagrams proportional the A -term of $(LH_u)^2$).*

the top-quark Yukawa coupling, respectively. Notice that the flavor structure of the RGE is dictated by how m and λ_E transform under $U(3)_L \otimes U(3)_E$ flavor rotations of the left-handed lepton doublets (L) and singlets (E). The first two terms on the r.h.s. of eq. (1) arise at the one-loop level and have been computed in [5]: these terms do not change the rank of m . From the explicit calculation of the first two-loop diagram in fig. 1 we have deduced the coefficient of the last term, which is the dominant effect that increases the rank of m . The dots denote other effects at two-loop order and higher, that do not give qualitatively new effects.

We parameterize the neutrino mass matrix in the e, μ, τ basis as $m = V^* \text{diag}(m_1, m_2, m_3) V^\dagger$ where m_1, m_2, m_3 are complex eigenvalues, $V = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}) \cdot \text{diag}(1, e^{i\phi}, 1) \cdot R_{12}(\theta_{12})$, and $R_{ij}(\theta_{ij})$ represents a rotation by θ_{ij} in the ij plane. ϕ is the CP-violating phase in oscillations. We denote the two leading eigenvalues of the neutrino mass matrix as m_a and $e^{-2i\alpha} m_b$ with $m_a > m_b$, and we denote as $e^{-2i\beta} m_c$ the smallest eigenvalue. α and β are the usual Majorana phases. If neutrinos have normal hierarchy, then $a = 3$ ($|m_3| = m_{\text{atm}} \approx 0.05 \text{ eV}$), $b = 2$ ($|m_2| = m_{\text{sun}} \approx 0.009 \text{ eV}$), $c = 1$ ($m_1 = m_{\text{min}}$). If neutrinos have inverted hierarchy, one instead has $a = 2$, $b = 1$ (neglecting the solar mass splitting, the two heavier neutrinos are degenerate with mass m_{atm}) and $c = 3$. In both cases, m_{min} has a Majorana phase, $m_{\text{min}} = |m_{\text{min}}| e^{-2i\beta}$. We can neglect the $\lambda_{e,\mu}$ couplings, such that $Y \simeq (0, 0, \lambda_\tau^2)$.

In the ‘diagonalize and run’ approach, eq. (1) can be converted into a RGE for the smallest eigenvalue m_{min} :

$$\frac{dm_{\text{min}}}{d \ln \mu} = \frac{2\lambda_\tau^4}{(4\pi)^4} \left[(V_{\tau c} V_{\tau a}^*)^2 m_a + (V_{\tau c} V_{\tau b}^*)^2 e^{-2i\alpha} m_b \right] + \dots \quad (2)$$

where we only wrote the two-loop terms that generate it.

The ‘run and diagonalize’ approach allows to write the explicit solution to eq. (1) in the charged-lepton eigenstate basis as:

$$m(\mu) = r \begin{pmatrix} m_{ee}^{(0)} & m_{e\mu}^{(0)} & y m_{e\tau}^{(0)} \\ m_{\mu e}^{(0)} & m_{\mu\mu}^{(0)} & y m_{\mu\tau}^{(0)} \\ y m_{\tau e}^{(0)} & y m_{\tau\mu}^{(0)} & y^2 z m_{\tau\tau}^{(0)} \end{pmatrix}, \quad (3)$$

where $m_{ij}^{(0)}$ are the initial values of the mass matrix (at some heavy scale where we assume $\det m^{(0)} = 0$) and

$$\ln r(\mu) = \int (\lambda - 3g_2^2 + 6\lambda_t^2) dt, \quad \ln y(\mu) = -\frac{3}{2} \int \lambda_\tau^2 dt, \quad \ln z(\mu) = \frac{2}{(4\pi)^2} \int \lambda_\tau^4 dt, \quad (4)$$

where $t = \ln \mu / (4\pi)^2$. Eq. (3) shows that one loop effects generate a fake m_{\min} if numerical inaccuracies or partial RGE resummation of higher orders terms break the $y \cdot y \neq y^2$ relation, mimicking the effect of the two-loop term z . Running down to the electroweak scale and computing the determinant, one gets the radiatively-generated light neutrino mass and its Majorana phase:

$$m_{\min} = (z - 1) \left[(V_{\tau c} V_{\tau a}^*)^2 m_a + (V_{\tau c} V_{\tau b}^*)^2 e^{-2i\alpha} m_b \right]. \quad (5)$$

Working at first order in $m_{\text{sun}} \ll m_{\text{atm}}$ and in $\theta_{13} \ll 1$, and inserting numerical best-fit values $\theta_{\text{atm}} = \pi/4$ and $\tan^2 \theta_{\text{sun}} = 1/2$ in the subleading terms, one gets

$$m_{\min} = (z - 1) \left[e^{2i\phi} \frac{m_{\text{atm}}}{4} \sin^2 2\theta_{\text{atm}} \sin^2 \theta_{\text{sun}} + \frac{m_{\text{sun}} e^{-2i\alpha} - 3\sqrt{2} m_{\text{atm}} \theta_{13} e^{i\phi}}{18} \right], \quad (6)$$

in the case of normal mass hierarchy, and

$$m_{\min} = (z - 1) e^{-2i(\phi+\alpha)} \left[\frac{m_{\text{atm}}}{4} \sin^2 2\theta_{\text{atm}} (\cos^2 \theta_{\text{sun}} + e^{2i\alpha} \sin^2 \theta_{\text{sun}} + \frac{2\sqrt{2}}{3} e^{i\phi} (e^{2i\alpha} - 1) \theta_{13}) \right] \quad (7)$$

in the case of inverted mass hierarchy. We performed a global fit of present oscillation data* finding that the term in square brackets in eq. (6) lies between 1.4 and 8 meV at 3σ confidence level. The analogous term for inverted hierarchy in eq. (7) lies between 0 and 16 meV. The RGE factor is

$$z - 1 \approx \frac{2}{(4\pi)^4} \frac{m_\tau^4}{v^4} \ln \frac{M}{M_Z} \approx 0.85 \cdot 10^{-12} \ln \frac{M}{M_Z} \quad (8)$$

where $v = 174 \text{ GeV}$; the numerical value, obtained from a numerical solution of SM RGE equations, agrees closely with the simple analytical approximation; M is the heavy scale where the initial condition $\det(m^{(0)}) = 0$ holds. For $M \lesssim 10^{14} \text{ GeV}$ we find $|m_{\min}| \sim 10^{-13} \text{ eV}$.

In practice, no significant physical effects arises in the limit $m_{\min} \rightarrow 0$,[†] so that such a small value of m_{\min} is not testable within the SM. For example, oscillation predictions for $0\nu 2\beta$ [6] are the same for any $m_{\min} \ll m_{\text{sun}}$.

3 Minimal Supersymmetric Standard Model

As shown in fig. 1, supersymmetry implies additional two-loop contributions to the neutrino mass matrix. The diagrams in fig. 1 are those surviving in the limit of exact supersymmetry with $\mu = 0$. In this limit their sums vanishes, as dictated by the non renormalization theorem that only allows corrections to wave-functions and therefore forbids a RGE effect that increases the rank of the neutrino mass matrix. Indeed RGE corrections have been computed up to two loop order [7] and the $Y \cdot m \cdot Y$ term is absent.

However, supersymmetry must be broken, presumably at the weak scale. Computing the diagrams in fig. 1, plus others with A -term vertices, one loses the large RGE logarithm present in the SM (the sum of the integrals is convergent) but gains a $\tan^4 \beta$ enhancement, because each one of the four τ Yukawa couplings λ_τ is enhanced by $\tan \beta$. The induced neutrino masses can still be expressed by eqs. (6)–(7) with the replacement of the RGE factor with

$$(z - 1) \rightarrow \frac{1}{(4\pi)^4} \frac{m_\tau^4}{v^4} \tan^4 \beta \cdot f(m_{\tilde{L}}, m_{\tilde{E}}, m_H, m_{\tilde{H}}) \quad (9)$$

* $\theta_{12}, \theta_{23}, |\Delta m_{23}^2|, \Delta m_{12}^2$ have been measured, there is an upper bound on θ_{13} , ϕ and α are unknown [1].

[†]The $\text{SU}(2)_L$ analogous of the QCD θ angle gives anyway negligible effects exponentially suppressed by $1/\alpha_2$.

(in the large $\tan\beta$ limit), where $v = 174\text{ GeV}$ and f is a finite adimensional order one function. The explicit form of f is not illuminating.

Furthermore, there is an additional effect [8] that gives a potentially dominant contribution of relative order $g^2 \ln^2(M_{\max}/m_{\text{soft}})/(4\pi)^2$ with respect to eq. (9): the SUSY breaking term $A_{ij}(\tilde{L}_i H_u)(\tilde{L}_j H_u)$ can contribute to the neutrino mass matrix via a gaugino-slepton loop. Depending on taste it can be either classified as a 1 or 2 or 3 loop effect. We assume that the soft terms are flavor independent at $M_{\max} = \min(M, M_{\text{med}})$, where M_{med} is the mediation scale of soft terms, equal to M_{Pl} in supergravity-mediated models. Slepton masses get corrected by flavor-dependent RGE effects; the eigenvectors of A_{ij} get rotated relative to those of m_{ij} and the rank of the A_{ij} -term matrix is increased already by one loop RGE-running between m_{soft} and M_{\max} . Indeed, the one loop RGE for $\hat{A}_{ij} \equiv v^2 A_{ij}/m_{ij}$ is

$$(4\pi)^2 \frac{d\hat{A}_{ij}}{d\ln\mu} = 2(\delta_{i\tau} + \delta_{j\tau})\hat{A}_\tau \lambda_\tau^2 + \dots \quad (10)$$

where $A_\tau = \hat{A}_\tau \lambda_\tau$ is the A -term of the τ -Yukawa coupling and \dots denotes other terms not crucial for the present discussion. The solution has the form

$$\hat{A}(\mu) = \begin{pmatrix} \hat{A}^{(0)} & \hat{A}^{(0)} & \hat{A}^{(0)} + \epsilon \hat{A}_\tau \\ \hat{A}^{(0)} & \hat{A}^{(0)} & \hat{A}^{(0)} + \epsilon \hat{A}_\tau \\ \hat{A}^{(0)} + \epsilon \hat{A}_\tau & \hat{A}^{(0)} + \epsilon \hat{A}_\tau & \hat{A}^{(0)} + 2\epsilon \hat{A}_\tau \end{pmatrix}, \quad \epsilon \simeq \frac{\lambda_\tau^2}{(4\pi)^2} \ln \frac{M_{\max}}{\mu}. \quad (11)$$

If $A_{ij}^0 = \hat{A}^{(0)} m_{ij}^{(0)}/v^2$ has one zero eigenvalue, the additive correction in eq. (11) transforms it into a small $\mathcal{O}(\epsilon^2)\hat{A}_\tau$ eigenvalue in A_{ij} , justifying our above estimate.[‡] Due to the large uncertainty (all sparticle masses are unknown), we just estimate the slepton-gaugino loop [8] contribution to be:

$$m_{\min} \sim m_{\text{atm}} \frac{g^2}{64\pi^2} \frac{\lambda_\tau^4}{(4\pi)^4} \frac{m_\chi \hat{A}}{m_{\text{soft}}^2} \ln^2 \frac{M_{\max}}{m_{\text{soft}}} \sim 10^{-10} \text{ eV} \cdot \left(\frac{\tan\beta}{10}\right)^4,$$

comparable to the 2 loop contribution of eq. (9).

4 Affleck-Dine leptogenesis

In the supersymmetric context, such a small neutrino mass can have phenomenological consequences. Indeed, recent analyses found that a scalar condensate along the LH_u flat direction can produce the observed baryon asymmetry if $m_{\min} \sim 10^{-(12\div 9)} \text{ eV}$ [3], where the uncertainty is due to our lack of knowledge about the reheating temperature, sparticle masses and CP phases.

The results of [3] cannot be immediately applied to our scenario because their neutrino masses are not radiatively generated. Nevertheless, let us first summarize some of the key points of [3]. As usual, the $B - L$ conserving sphalerons transfer a lepton asymmetry into a baryon asymmetry: this singles out the LH_u direction. It develops, during inflation, a large scalar

[‡]We here elaborate on the possibly surprising claim that one-loop RGE running generates no $\mathcal{O}(\epsilon)$ eigenvalue and generates a two-loop-like $\mathcal{O}(\epsilon^2)$ eigenvalue. Notice that the structure of the additive terms to A_{ij} in eq. (11) (0 or 1 or 2 depending on how many τ there are in ij) is exact, such that we do control $\mathcal{O}(\epsilon^2)$ terms. Furthermore, our result is compatible with the general statement [9] that RGE effects in softly broken supersymmetry can be condensed into a renormalization of the superfields, with renormalization factors and couplings appropriately promoted to spurion superfields. In our case it (roughly) means that at one loop $\tilde{m}_{ij} \equiv m_{ij} + \theta\theta A_{ij} + \dots$ only gets corrected via a $\tilde{y} \approx y + \theta\theta\epsilon A_\tau + \dots$ multiplicative renormalization of the L_τ superfield, where y is MSSM analogous of SM y in (4). The vanishing of $\det \tilde{m}$ implies the vanishing of $\det m$ and of a combination of $A \times m$ (verified by eq. (11)), and does not imply the vanishing of $\det A$.

condensate $\varphi \equiv \langle \tilde{L} \rangle \simeq \langle H_u \rangle$ if φ has a soft mass² of order $-H^2$ from the inflationary vacuum energy, where H is the expansion rate during inflation, then one expects $\varphi \sim v\sqrt{H/m_{\min}}$. The smallest neutrino mass m_{\min} gives the dominant effect because it allows the largest φ . When H^2 decreases below the mass² terms in $V(\varphi)$ (the normal soft masses, m_{soft}^2 , plus thermal corrections of $\mathcal{O}(T^2)$, relevant if the reheating temperature after inflation is high enough) the condensate starts to oscillate generating a sufficient lepton asymmetry,[§] because the potential contains a φ^4 term that breaks lepton number, coming from the A -term of the $(LH_u)^2$ operator.

Let us now come to the case of radiatively-generated m_{\min} . The dynamics of AD leptogenesis is not directly controlled by neutrino masses, but by the φ^4 and $|\varphi|^6$ terms in the $V(\varphi)$ potential at the beginning of oscillations, respectively generated by the A -term and by the F -term of the neutrino mass operator $(LH_u)^2$. The φ^4 term acts as the source of lepton-number breaking and the φ^6 term limits the initial vev of φ . Denoting by r_4 and r_6 the correction factors of these terms with respect to the ‘standard’ values considered in earlier analyses [3], the final amount of baryon asymmetry gets corrected by r_4/r_6 .

We therefore need to compute r_4 and r_6 . For simplicity we assume that φ remains below the scale of new physics that generates the LH_u operator.[¶] The previous section suggests that, similarly to the lightest neutrino mass, the φ^4 and the $|\varphi|^6$ terms in $V(\varphi)$ are generated by quantum corrections, such that today (after the end of inflation, at temperature $T \ll m_{\text{soft}}$) $r_{4,6}$ are not much different from one. However, quantum corrections depend on sparticle masses, which had different values during the epoch relevant for AD leptogenesis. There are various effects. Corrections to soft terms of order H^2 (inflationary masses) and of order T^2 (thermal masses) do not qualitatively change our results, because they generically break supersymmetry. The time dependence of φ provides one more source of SUSY-breaking via the D -terms; furthermore, $\varphi(t)$ directly contributes to $V(\varphi)$ when inserted into higher dimensional D -terms such as $(L\partial H_u)^2/M^3$. On the contrary, the large vev φ , inserted in the $\lambda_\tau LEH_d$ coupling, generates a large supersymmetric mass $\sim \lambda_\tau \varphi$ for the E and H_d^- particles and sparticles. This suggests that $r_4, r_6 \sim m^2/(\lambda_\tau \varphi)^2$ where m^2 are SUSY-breaking masses coming from the effects discussed above. Then r_4/r_6 remains of order one, such that AD leptogenesis remains successful for the standard value $m_{\min} \sim 10^{-9 \div 12}$ eV. We have shown that a value in this range does not need contrived flavor models and can be naturally generated by quantum corrections. This encouraging result might be tested in a more stringent way if sparticles will be discovered, and if their masses and especially $\tan\beta$ will be measured.

5 Conclusions

Assuming that two neutrinos have Majorana masses and that the lightest neutrino is massless at tree level, we computed the mass generated by quantum corrections, and its Majorana phase. In the SM two loop RGE running gives $|m_{\min}| \sim 10^{-13}$ eV. In the MSSM supersymmetry breaking generates various flavor matrices that contribute in different ways; the typical result is $|m_{\min}| \sim 10^{-10}$ eV $(\tan\beta/10)^4$, enhanced by four powers of $\tan\beta$. Such a small neutrino mass is compatible with the restricted range of values that allows successful Affleck-Dine leptogenesis along the LH_u flat direction.

[§]Unless the system remains trapped in one of the unphysical vacua often present in the MSSM; thermal effects allow to partially predict which local minimum is dynamically selected as the vacuum [10].

[¶]In the see-saw scenario, this new physics are right-handed neutrinos of mass M , with $M < 10^{14 \div 15}$ GeV if we want to remain in a perturbative regime. It is not clear to us what happens if instead $\varphi > M$; possibly φ would slide up to the GUT scale (around 10^{16} GeV) rather than being limited by the $|\varphi|^6$ term, giving rise to a dynamics somewhat different from the one studied in [3].

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